Light Out-Coupling for Reflective Displays: Simple Geometrical Model, MATLAB Simulation, and Experimental Validation

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***Abstract*—Light out-coupling efficiency is a common issue for light emissive displays and devices, but is rarely discussed for reflective displays (e-Paper). The latest generation of reflective displays include light scattering particles and reflectance that is close to Lambertian (paper-like). However, this light scattering inside a reflective pixel can lead to total-internal reflection and additional light loss. This paper provides both a simple geometrical and a more detailed MATLAB simulation for calculating light out-coupling. Both approaches were found to agree with experimental results that reveal 5%–20% optical loss due to inefficient out-coupling. These out-coupling models improve the accuracy of predicting optical performance in reflective displays.**

***Index Terms*—Light out-coupling, displays, scattering, total-internal reflection.**

I. INTRODUCTION

ELECTRONIC displays now exhibit impressive performance and cost maturity in both hand-held and large-area applications like television. Now emerging rapidly is reflective displays, or more commonly referred to as electronic paper (e-Paper) [1]. Compared to emissive [organic light-emitting display (OLED)] and transmissive [liquid crystal display (LCD]) displays: e-Paper has several distinct advantages such as low power, less eye and weight fatigue (lighter battery), contrast-ratio that is as good in direct sunlight as it is indoors, and arguably the most adaptable technology for rollable displays [2]. However, unlike emissive and transmissive displays, limitation in visual brightness of e-Paper technology cannot be overcome by simply increasing lighting or electrical power. There are numerous optical losses [1] including absorption, pixel border, inefficient reflection, Fresnel reflection, etc. It is generally desired that e-Paper be Lambertian in appearance (i.e., paper-like, the contrast is the same regardless of viewing or illumination angles). As shown in Fig. 1(a), a simple specular reflective pixel exhibits a mirror like reflection which results in poor visual performance but no issue with light out-coupling. To scatter the light and achieve a more diffuse (semi-Lambertian) reflection, one can implement one of several techniques: Fig. 1(b)—a transmissive light valve and rear diffuse reflector; Fig. 1(c)—a rear diffuse electrode; Fig. 1(d) a self-diffuse pixel; not-shown, a front diffuser film (light scattering). From the perspective of contrast-ratio, a front diffuser or rear diffuse reflector [see Fig. 1(b)] are less desirable because of increased reflections that do not contribute to pixel performance. An approach that internally scatters light might therefore be preferred, however, this approach [see Fig. 1(c) and (d)] presents an additional challenge in light out-coupling. Inside the pixel, light is scattered to a wider distribution of angles, some of which are totally internally reflected at the front air interface. This total-internal reflection would not be an issue if the pixel were without optical loss, but in practice, the reflective materials inside the pixel are not perfect. A portion of the totally internally reflected light is absorbed before it is rescattered and out-coupled (Fig. 1(e)) [3]. The question is then asked, does this light out-coupling play a significant role in self-diffuse e-Paper technologies like electrophoretic [4], electrochromic [5], liquid powder [6], and electrofluidic [7]? This paper answers that question, through both a simple geometrical and a more detailed MATLAB simulation for calculating light out-coupling in e-Paper. Both approaches were found to agree with experimental results that show a significant 5%–20% optical loss due to inefficient out-coupling. Both model and experiment predict a strong dependence on reflector efficiency and refractive index of the material adjacent to or surrounding the reflector material. The out-coupling model presented herein improves the accuracy of predicting optical performance in e-Paper, and shows that light out-coupling is an issue that cannot be ignored.

II. EXPERIMENT

First, the experimental measurement setup is described. These measurements were made to allow comparison to the theoretically predicted data. Generic reflective test samples were fabricated in order to mimic the basic construction of many e-Paper pixels (Table I). Indium-tin-oxide (ITO) coated glass was used as the transparent front substrate. A SU-8 polymer layer (MicroChem 2000) of thickness 4.5 μm was spin coated onto the ITO to mimic a dielectric or alignment layer, or other possible coating. For the lower substrate, two reflective options were explored: (1) semi-diffuse rough Aluminium on glass (2) diffuse white PET film (loaded with TiO_2 particles).

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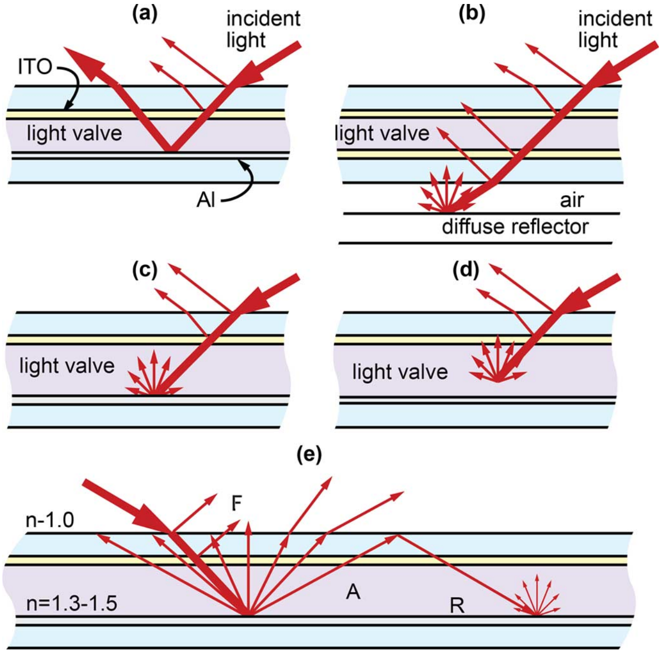

Fig. 1. (a)–(d) Typical reflective modes in e-Paper technologies: (a) specular; (b) rear diffuser; (c) rough or wavy Al; and (d) self-diffuse. (e) Simplified diagram of light out-coupling—inefficient light out-coupling.

TABLE I
OPTICAL STACK PARAMETERS FOR VISIBLE LIGHT

Layer	n	R (reflectivity)	D (diffusivity)
Glass	1.5	8.6%	0
ITO	1.9		0
SU-8	1.6	0	0
Fluids	1-1.5	0	0
Reflector		Al – 0.79 / PET - 0.89	Al - 0.55 / PET - 0.86

A clear fluid was then sandwiched between the substrates in a gap of 70 μm. This clear fluid was used to simulate the refractive index of the active layer, and ranged from $n = 1$ to 1.47. Test fluids included, GE silicone oil SF1555 ($n = 1.47$), Tetradecane ($n = 1.43$), Isopropyl Alcohol ($n = 1.38$), DI water ($n = 1.33$), DuPont Vertrel XF ($n = 1.24$), and also no fluid (air, $n = 1$). Some pixel active layers, such as those used for liquid crystals or high density liquids for stably dispersing particles, can be even higher in refractive index.

Reflection measurements were performed with a LabSphere RT-060-SF integrating sphere with four ports: a top port for white light introduction from a Thorlabs OSL Fiber illuminator (including a baffle to prevent direct light incidence on the sample), a 9° port for an optical fiber leading to an Ocean Optics HR4000CG-UV-NIR spectrometer, a -9° port to exclude specular reflection (if not used, a diffuse reflector was placed at the port), and a sample port where the test substrates were placed. Uncollimated light from a tungsten lamp was input to the top port and equally distributed to all other points of the inner sphere, including the sample port, by multiple scattering reflections. Light reflected at 9° from the sample surface is then collected by an Ocean Optics P200-2-VIS-NIR optical fiber fitted with a 74-VIS collimating lens at the 9° detector port. For more detailed introduction, to the operation of integrating spheres the reader is directed to "A Guide to Integrating Sphere Theory and Applications" available through the LabSphere website [8].

III. FIRST THEORETICAL APPROACH: SIMPLE GEOMETRICAL MODEL

Both a simple geometrical model and a more sophisticated MATLAB simulation were developed for predicting light out-coupling. First, the simple geometrical model is described. The simple geometrical model includes only dominant optical factors (Fig. 1(e)) including: 1) the front substrate Fresnel reflection (F) and 2) optical absorption as light travels through semi-transparent layers, or as is absorbed at pixel borders and other lossy features $(1 - A)$, loss due to the internal reflector (R). The geometrical model also considers internal light scattering and the resulting total-internal reflection at the top-substrate/air interface. The cumulative effect of all these factors is best explained through a series approximation of the light out-coupling process.

First, the incident light traverses the front substrate/air interface and other interfaces with mismatched refractive index. A significant portion ($\sim$5%–10%) of the light is Fresnel reflected (F) such that the remaining light entering the pixel is $(1 - F)$. Next, this light reaches the rear diffusely reflecting surface and is reflected with an efficiency (R). While traversing various layers, there is also light loss due to optical absorption which only allows a fraction (A) to pass through. At this point, the amount of light reflecting back to the top substrate/air interface is therefore

$$(1 - F)RA. \tag{1}$$

Now, because the rear reflecting surface is diffuse and redistributes the light, a fraction of the light (P) will get into the air with the rest being totally internally reflected [see Fig. 1(e)]. Total internal reflection only occurs when the light strikes the air/glass interface beyond the critical angle. P can therefore be calculated as $1/n^2$ (see supplement file for derivation). As a result, after one cycle the total light out-coupling (LO_1) can be expressed as

$$\mathrm{LO}_1 = (1 - F)RAP. \tag{2}$$

Now, the total internally reflected light will again undergo absorption A and will be redistributed in angle by the diffuse reflector with an efficiency R. This light will reach the front substrate/air interface a second time and a percentage P will be out-coupled. This second light out-coupling contribution (LO_2) can then be expressed as

$$\mathrm{LO}_2 = \mathrm{LO}_1 \times RA(1 - P). \tag{3}$$

The process repeats such that the total light out-coupling is

$$\mathrm{LO} = \mathrm{LO}_1 + \mathrm{LO}_2 + \mathrm{LO}_3 + \mathrm{LO}_4 \ldots \tag{4}$$

Equation (4) can be rewritten with a power series approximation [9], and then re-including the original Fresnel reflected light (F) the total light out-coupling is

$$\mathrm{LO} = F + \frac{(1 - F)RAP}{1 - RA(1 - P)}. \tag{5}$$

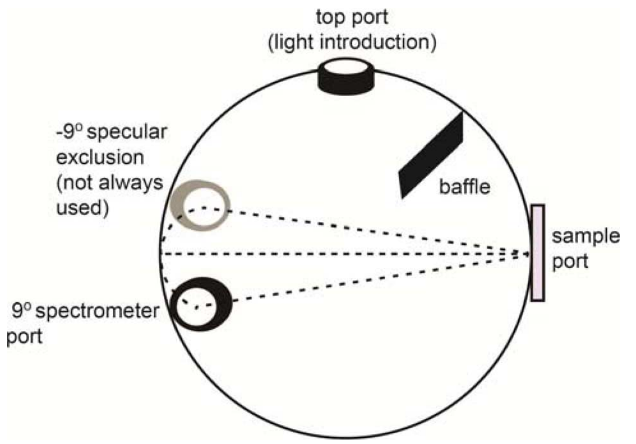

Fig. 2. Diagram of integrating sphere setup used for reflectance measurement.

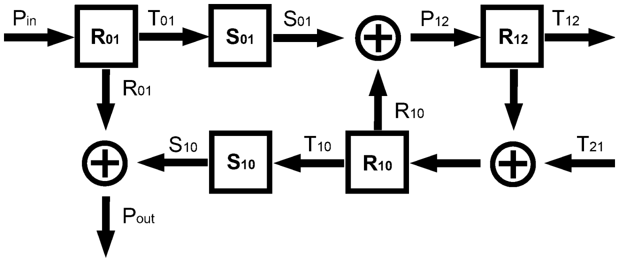

Fig. 3. MATLAB simulation flow. P_{in} is the incident power as a function of angle from normal. R_{ij} function block outputs the reflection and transmission, R_{ij} and T_{ij}, during transitions between interfaces. S_{ij} shifts the angle of the power function according to Snell's law, S_{ij}. P_{out}, the output power, is read as the sum of returned power.

All relevant parameters F, R, A and P can be measured and calculated to predict light out-coupling according to (5). The model does not include small factors such as thin-film interference and other difficult to measure absorptive or reflective losses. If desired, these losses can be all grouped within the parameter A, with A being fitted by the experimental data. This fitting approach may be particularly useful for complex pixel structures where the small optical losses are numerous and cumulatively can lead to a significant total optical loss.

IV. SECOND THEORETICAL APPROACH: MATLAB SIMULATION

A second more rigorous theoretical approach was developed with a MATLAB simulation. The simulation calculates the total percentage reflected light. Fresnel reflections and refractions from all optical layers are included. The MATLAB program flow is provided in the system diagram as shown in Fig. 3. Light in each layer is represented as distribution of power/rad, $P_{ij}(\theta)$, over angles off normal from 0° to 90°. The full equation for Fresnel refraction can be found elsewhere [10], and here the percentage reflection between adjacent layers i and j will be represented as $F_{ij}(\theta)$. The total reflected light at the interface between adjacent layers is:

$$R_{ij}(\theta) = F_{ij}(\theta)\begin{pmatrix}(P_{ij}(\theta) - F_{ij}(\theta))(1 - D_j)+\\ \mathrm{avg}(P_{ij}(\theta) - F_{ij}(\theta))D_j(\theta)\end{pmatrix} \tag{6}$$

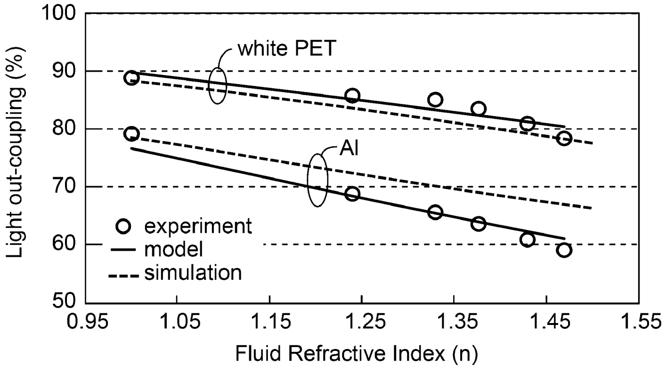

Fig. 4. Light out-coupling simulation and model results for aluminium and white PET substrate vs. refractive index of the light valve layer [Fig. 1(e)].

where $\mathrm{R_j}$ and D_j are the reflectivity and diffusivity of layer j. The transmitted light from any layer i to an adjacent layer j can be expressed as

$$T_{ij}(\theta) = 1 - R_{ij}(\theta). \tag{7}$$

According to Snell's law there will be a shift of the angle of the light as it traverses the interface between layers i and j which can be expressed by S_{ij} as

$$S_{ij}(\theta) = P_{ij}\left(\theta + \sin^{-1}\left(\frac{n_i}{n_j}\sin(\theta)\right)\right). \tag{8}$$

In order to find the power distribution of the diffused light we must integrate around the circumference of the hemisphere at each angle where $R = r\sin(\theta)$ so the light diffusing function can be expressed as

$$D_{ij}(\theta) = D_j\cos(\theta)\sin(\theta). \tag{9}$$

The input power to the simulation was set to a single angle of 9° off normal to match the integrating sphere setup in the experiment (Fig. 2). The input power can be expressed as

$$P_{\mathrm{in}} = \delta(9^\circ) \tag{10}$$

where δ is the impulse function such that all power is concentrated at this angle. The output will then be a fraction of 1 which can then be converted to a reflection percentage. Optical parameters were fed into the model as detailed in Table I. The full MATLAB simulation code is provided with the online supplementary material.

V. EXPERIMENTAL AND THEORETICAL RESULTS

Plotted in Fig. 4 are both the simple geometrical model (solid line), the MATLAB simulations (dashed line), and the experimentally measured data (circles). Both the simulation and model are in decent agreement with the experimentally measured results. First, as can be seen for the rough Al, up to 20% change in light out-coupling is observed as refractive index of the fluid is increased. For $n = 1.47$ the losses for one cycle of light were calculated as $F = 8.6\%, A = 94.26\%, R = 78.9\%, P = 46.28\%$, which after several cycles only allows ∼60% of the light to be reflected. If the internal reflector were lower efficiency (as is the case for many e-Paper technologies) then the light out-coupling efficiency could be even lower. Next consider the white PET measurements and simulation/model results. As expected, for all ranges of refractive index the out-coupled light is better for the white PET $(R = 89\%)$ than for the rough Al. Also expected for white PET, there is less of a decrease in out-coupling with increasing refractive index. A more efficient reflector will allow multiple light out-coupling cycles $(\mathrm{LO}_1, \mathrm{LO}_2, \mathrm{LO}_3$ etc...) to eventually out-couple the light without significant optical loss between each cycle. This assumes that A is low, which is the case for the experimental results discussed herein.

There are two imperfect aspects of the predicted and experiment results. First, the data for white PET is unusual when considering that the scattering does not take place in the fluid. The diffuse redistribution of the light occurs inside the white PET, where the refractive index is constant. This unexpected result might be explained by a large fraction of scattering that occurs near the surface of the PET (within a visible wavelength, or so, of the fluid). Second, for the rough aluminum, a discrepancy arises between the MATLAB simulation and the model/experimental results. This discrepancy may be due to the way in which the surface was modeled. It is well known that the reflection function of a rough surface differs greatly from a true Lambertian reflector. It is clear that the model and simulation results provided herein are less accurate for surfaces that are not closely Lambertian.

VI. ADDITIONAL EXPERIMENT ON VARIABLE DIFFUSE REFLECTIVITY

The experimental results discussed for Fig. 4 highlighted some of the differences between a Lambertian reflector such as white PET and a semi-diffuse (non-Lambertian) reflector such as rough Al. It is expected that a less Lambertian reflector will have improved out-coupling (at the cost of undesirable increase in mirror-like appearance). Using the experimental setup described in the previous sections, an additional set of experiments included five ranges of diffuseness for the reflection (Fig. 5). The experimental setup used a specular (flat) Al reflector of $R = 93.8\%$(450–650 nm averaged) with 0 to 5 layers of 3 M Scotch Magic™ tape over the Al surface, and above that fluid (water) and the front SU-8/ITO/glass substrate (Table I). Although the use of tape might be seen by some as unprofessional, it was determined highly useful for this work because: (1) it is optically non-lossy (the tape in the roll form has no coloration except white); (2) it is only semi-diffuse; (3) it is readily available for use by others in similar experimentation.

The data recorded in Fig. 5 was similar to all other experiments (regular lines), except the specular reflection was excluded for half the measurements (lines with circles) by placing a black surface at the -9° specular exclusion port. As shown in Fig. 5, for 1 layer of tape the reflectivity drops from 92% to 82%, due to inefficient light out-coupling. 1 layer of tape is still only about half diffuse and half specular (see solid versus dotted lines). Increasing the layer of tapes to 5 only decreases the reflectivity to 73%. The biggest drop in reflectivity was when the first layer of tape was added, therefore it appears that light out-coupling is a significant concern even for only semi-diffuse reflectors. It can, therefore, be further concluded that increasing the diffuseness of reflection is likely worthy in e-Paper pixels, even as out-coupling decreases slightly, because the appearance becomes more paper-like. This, of course, assumes the case of diffuse illumination as used herein. There is little difference in the data between 3 and 5 layers of tape, suggesting that the reflection inside the pixel is almost maximally diffuse after the 3rd layer of tape is added. For 3 to 5 layers of tape, the specular component of the reflection is still about 20%, and is likely due to Fresnel reflection from the layered tape itself and from other layers in the device. The increased diffuse reflection caused by the tape itself was measured to be ∼2% per layer of tape by measuring the tape over a black surface.

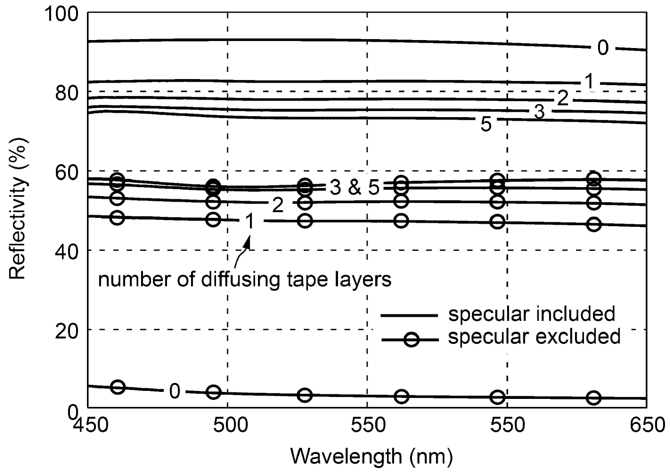

Fig. 5. Reflectivity measured as a function of diffuseness of the reflector layer. The measured stack was glass/Al/tape/water/SU-8/ITO/glass, with 0 to 5 layers of diffuse tape used to vary the diffuseness of the reflection. Both specular included (SI, regular lines) and specular excluded (SE, lines with circles) data was measured by use of the specular exclusion port shown in Fig. 2.

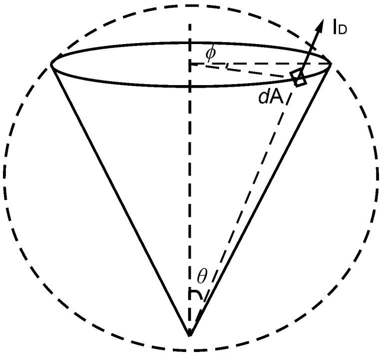

Fig. 6. Sphere of Lambertian reflection.

Although 2% per layer may seem small in terms of reflectivity, this increase is one reason why use of front-diffusers is not optimal when considering display contrast ratio.

VII. CONCLUSION

This paper both theoretically and experimentally shows that light out-coupling cannot be ignored in e-Paper devices. Because the out-coupling losses are compounded, light-out coupling is strongly dependent on the reflectivity inside the pixel. Light out-coupling can be ignored for devices that use a rear diffuser behind the pixel, but this approach is limited in maximum resolution and typically will have increased specular reflection [1]. These out-coupling models provided herein can be used to improve the accuracy of predicting optical performance in reflective displays. The experiments also highlight the need to report measurement of e-Paper devices with diffuse lighting sources.

APPENDIX
SUPPLEMENTARY DERIVATION

In this derivation, it is assumed that the reflections are all Lambertian, where the light intensity (I_D) for any direction (θ) follows Lambertian cosine law as:

$$I_D = I_0 \cos(\theta)$$

where I_0 is the incident light intensity normal to the reflective surface. The total light L within a cone (open angle 2θ) can be calculated as

$$L = \int I_D \cdot dA$$

Where $dA = r^2 \sin\theta \, d\phi \, d\theta$ is the area on the cap of the cone. Combining these equations we have:

$$L = \int_0^{\theta} \int_0^{2\pi} I_0 r^2 \cos(\theta) \sin(\theta) \cdot d\phi \cdot d\theta = \pi I_0 r^2 \sin^2(\theta)$$

Because the reflected light is diffusely reflected within a medium of refractive index $n < 1.5$, and because the other adjacent layers all have refractive >1.5, there is no total internal reflection until the light hits the interface with air, which is typically glass/air. Assume the glass refractive index is n_g. Then the largest incident angle at the interface is $\theta_{\mathrm{imax}} = \sin^{-1}(n/n_g)$, where n is the refractive index of the medium where the diffuse reflection occurs. The critical angle at glass/air interface is $\theta_c = \sin^{-1}(1/n_g)$. Light with an incident angle beyond this value will be reflected back into glass. Therefore the out-coupled light fraction P can be calculated as:

$$P = \frac{L_c}{L_{i\,\max}} = \frac{\sin^2(\theta_c)}{\sin^2(\theta_{i\,\max})} = \frac{1}{n^2}$$

This $1/n^2$ out-coupling fraction is the same as that used for calculating light out-coupling in emissive devices such as LEDs.

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